FPGA Based Acceleration of Computational Fluid Flow Simulation on Unstructured Mesh Geometry

Zoltán Nagy, Csaba Nemes, Antal Hiba, András Kiss, Árpád Csík, Péter Szolgay

Computer and Automation Research Institute

Pázmány Péter Catholic University

Széchenyi István University

Introduction

- Challenges in numerical solution of Partial Differential Equations
- High performance Computational Fluid Dynamics (CFD) accelerator
- Arithmetic unit generation and optimization
- Off-chip data access optimization
- Results
- Conclusions, future work

Acceleration numerical solution of Partial Differential Equations (PDE)

- Wide variety of physical phenomenon
  - sound, heat, elasticity, electrodynamics or fluid flow
- Computational fluid dynamics (CFD)
- Discretization over fine mesh
  - 5M mesh points - Air flow simulation around a car or airplane
  - 200M mesh points – jet engine acoustic modeling
- Weeks of simulation time on clusters
  - Low processor utilization ~10%
  - Weak scalability over ~100 nodes

Workflow
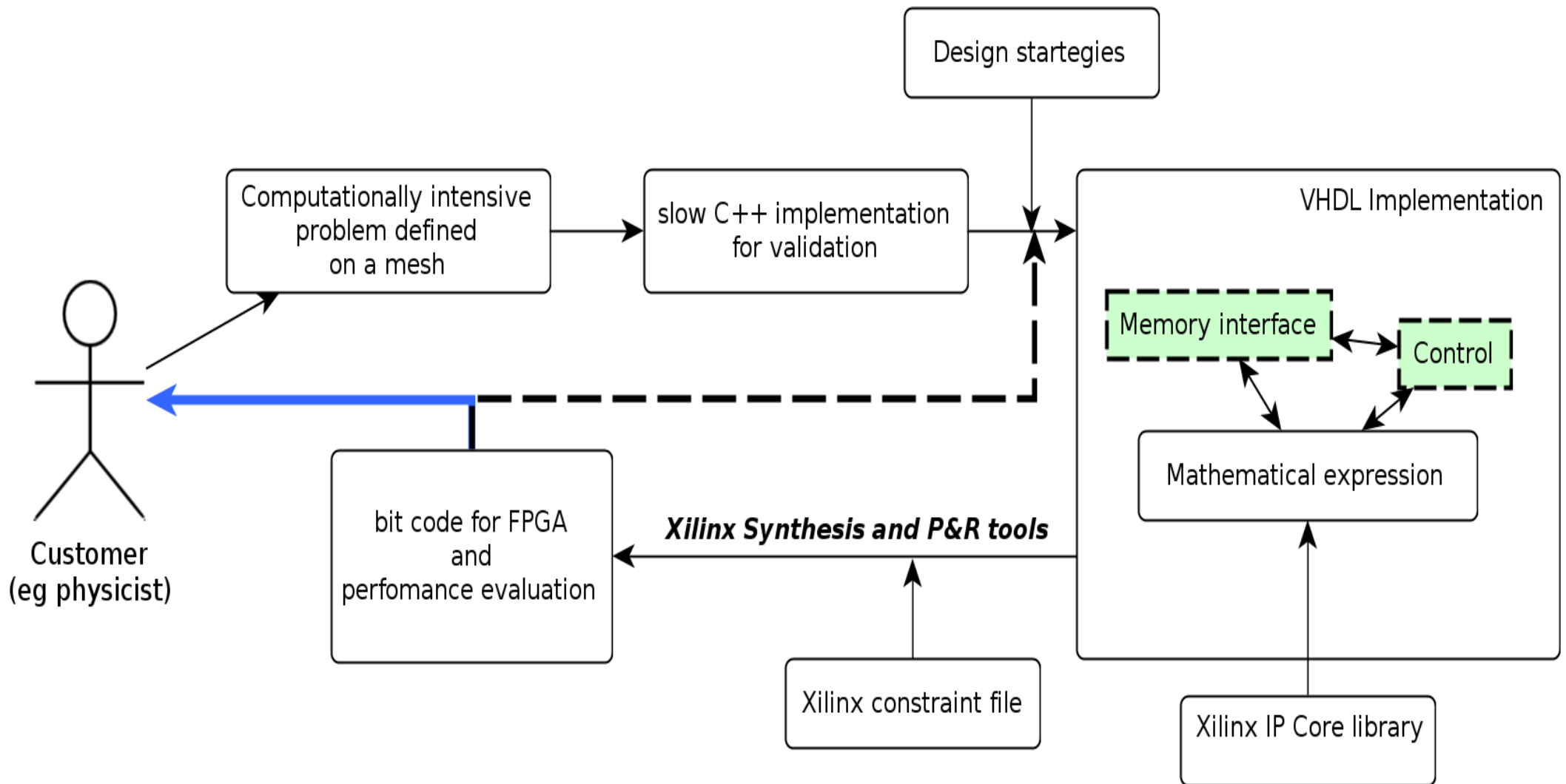
Inviscid, Adiabatic, Compressible flows

- Euler equations:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla\left(\rho vv + \hat{I}\, p\right) = 0$$

$$\frac{\partial E}{\partial t} + \nabla\left((E + p)v\right) = 0$$

- Total energy is defined as:

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho v \cdot v$$

- Notations:
  - t: time
  - ∇: Nabla operator
  - ρ: density
  - v(u, v): velocity vector field
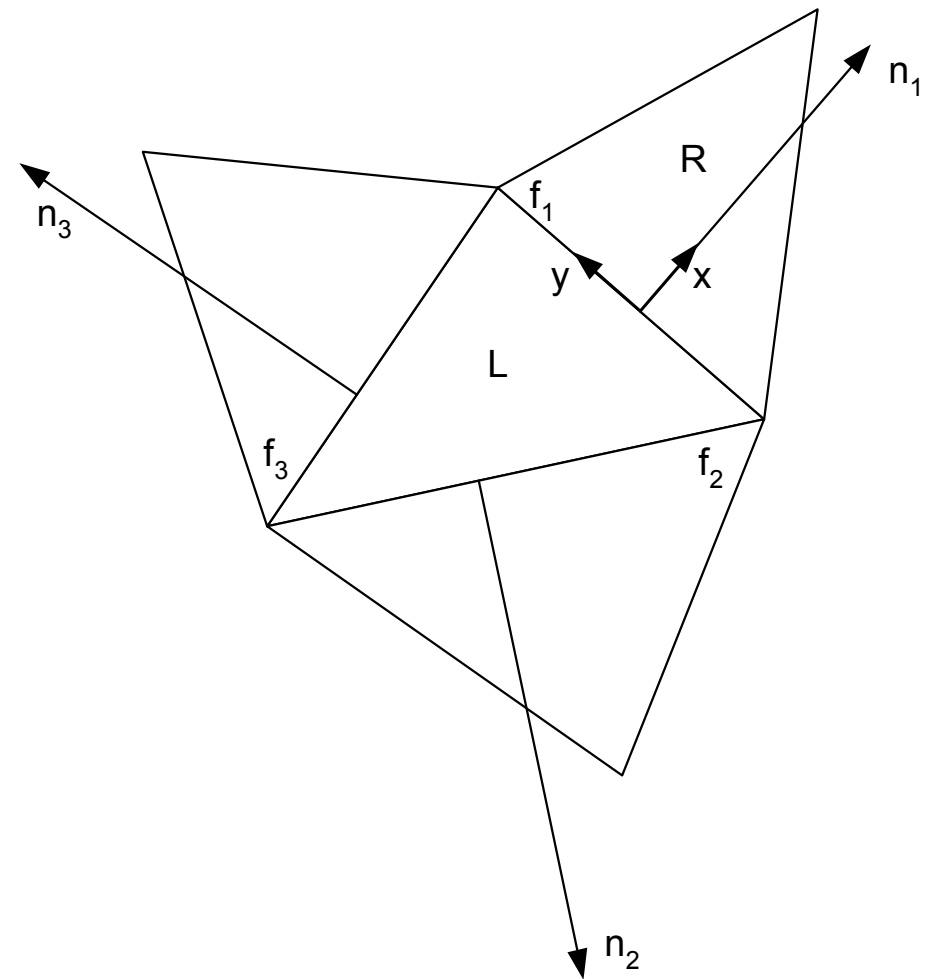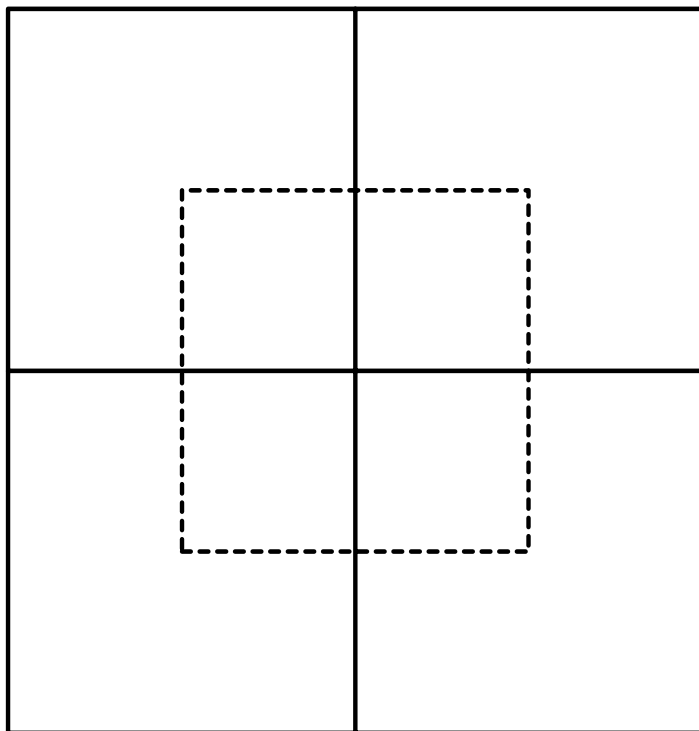 - p: pressure
  - I: identity matrix
  - E: total energy
  - γ: ratio of specific heats
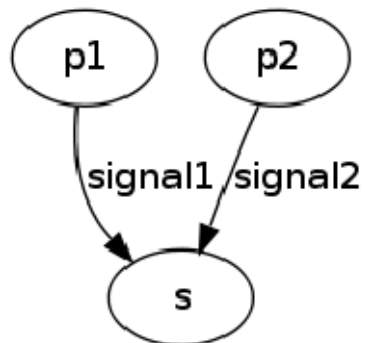
First order Lax-Friedrichs approximation

$$U_{i,j}^{n+1} = U_{i,j}^{n} - \frac{\Delta t}{V_{i,j}} \sum_{f} \mathbf{F}_{f} \cdot \mathbf{n}_{f}$$

$$F^{N} = \frac{F_L + F_R}{2} - \left(\left|\overline{u}\right| + \overline{c}\right)\frac{U_R - U_L}{2}$$

Data-flow model

```
1. ...
2. Prod<INSIZE,INSIZE,OUTSIZE> p1,p2;
3. Sum<INSIZE,INSIZE,OUTSIZE> s1;
4. ...
5. p1− >c(signal1);
6. p2− >c(signal2);
7. s1− >a(signal1);
8. s1− >b(signal2);
9. ...
```
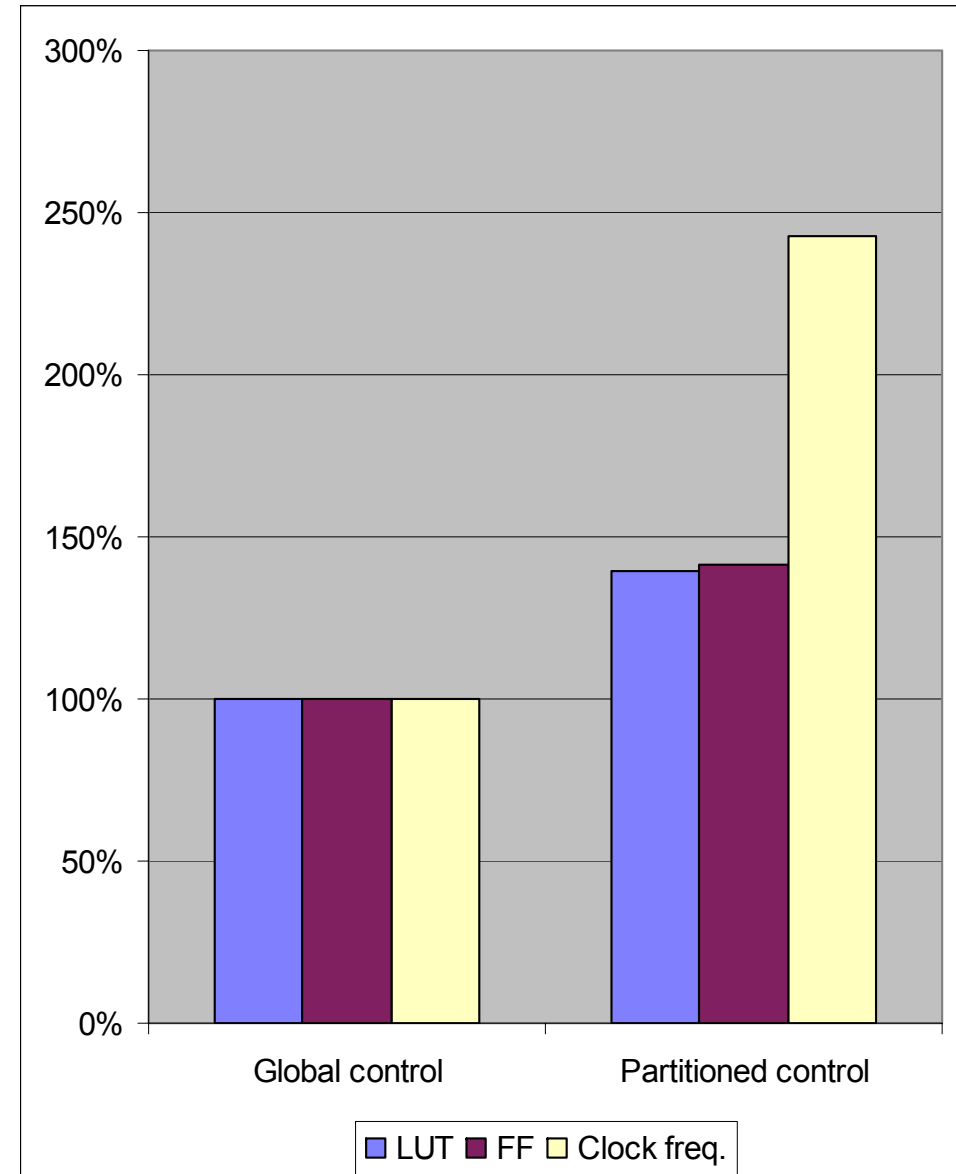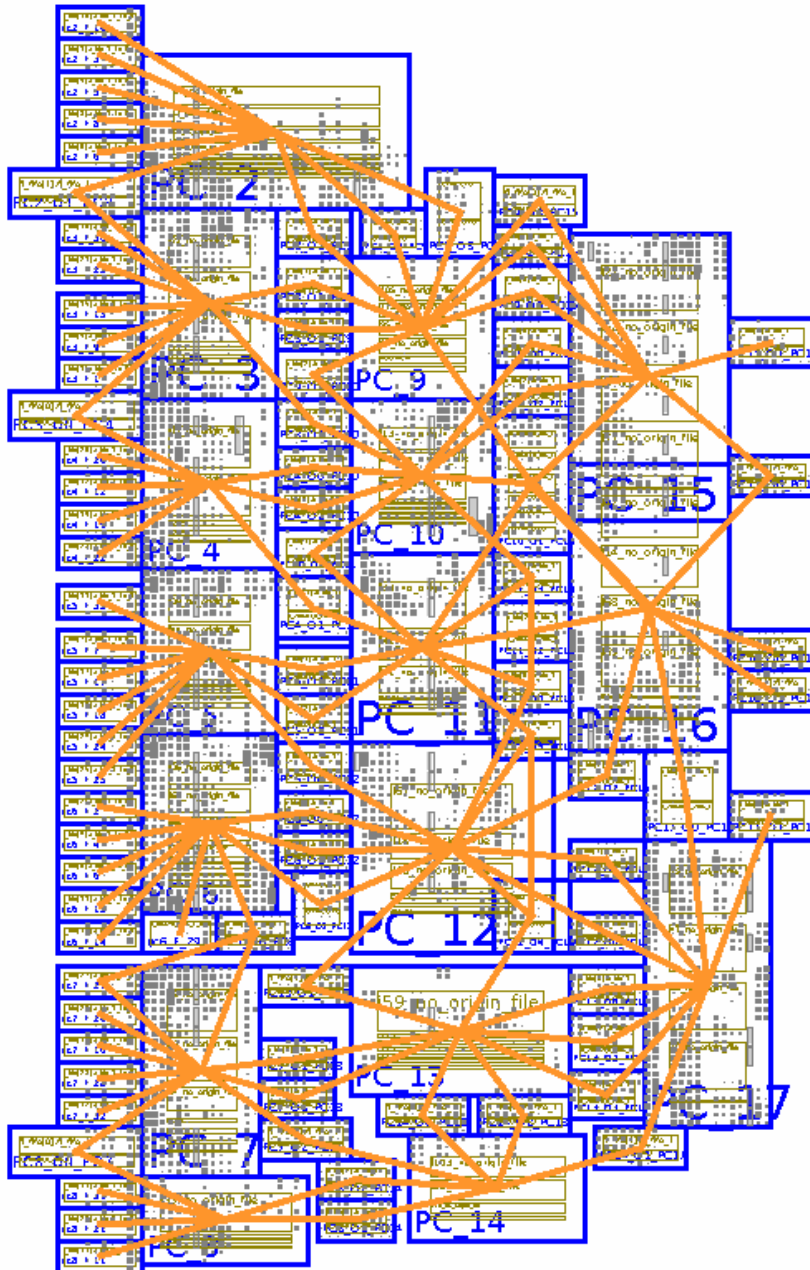

- Mathematical expression implemented in SystemC is converted to a hypergraph.
- Nodes = arithmetic units
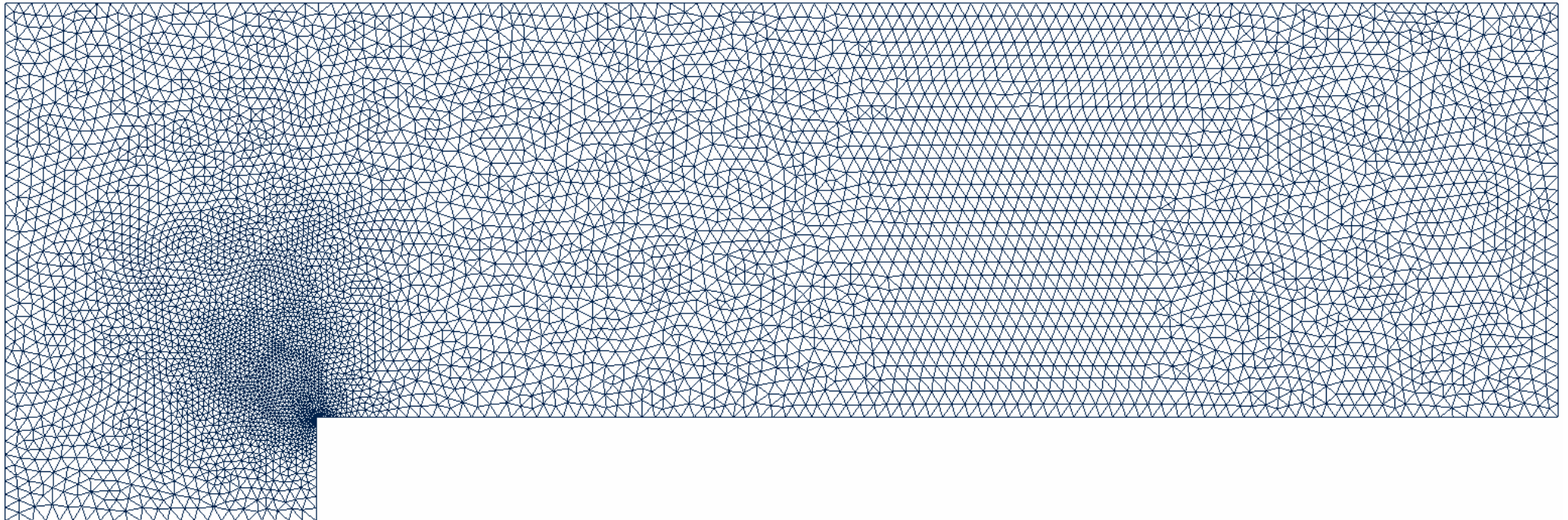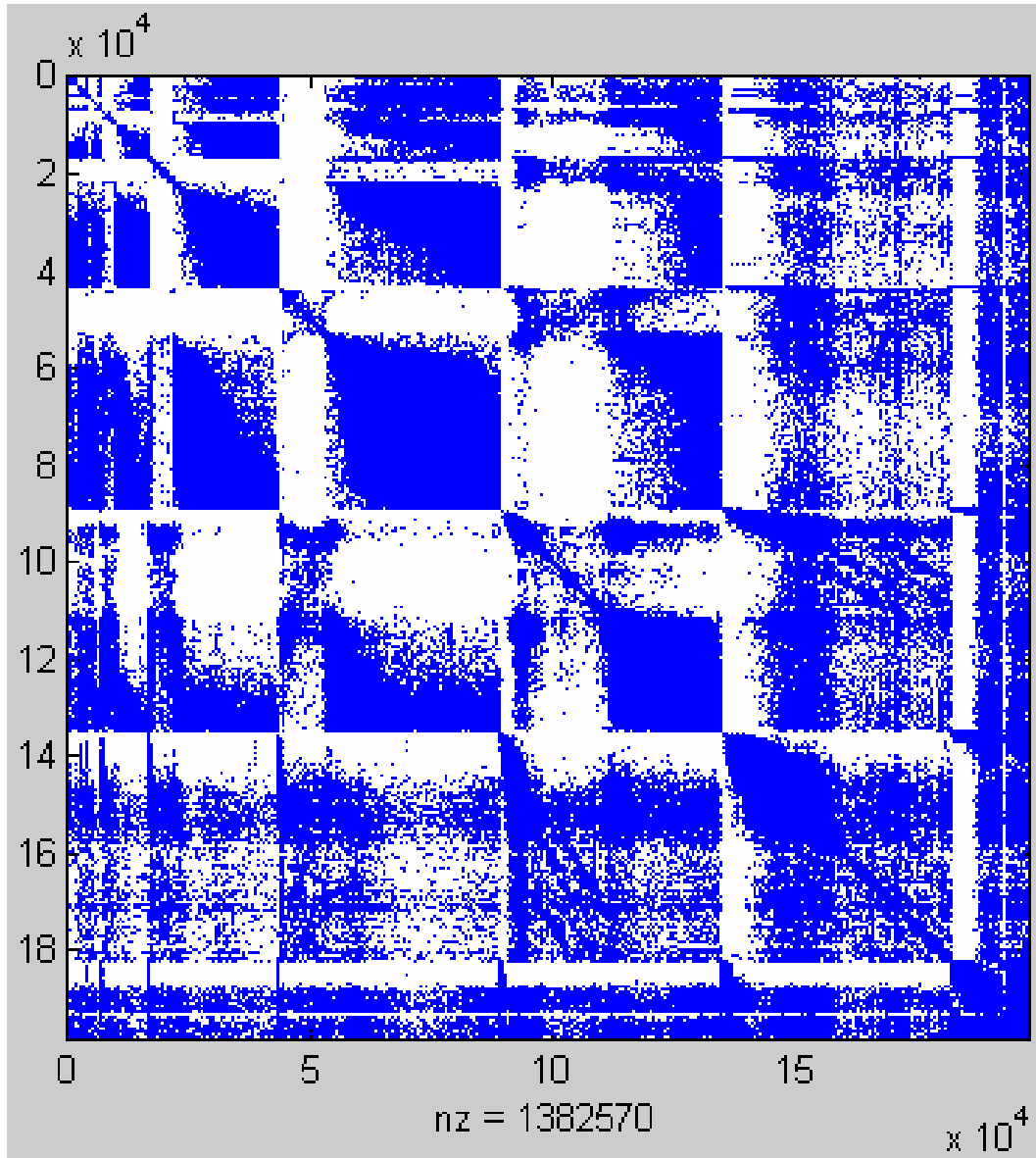- Hyperarcs = connections

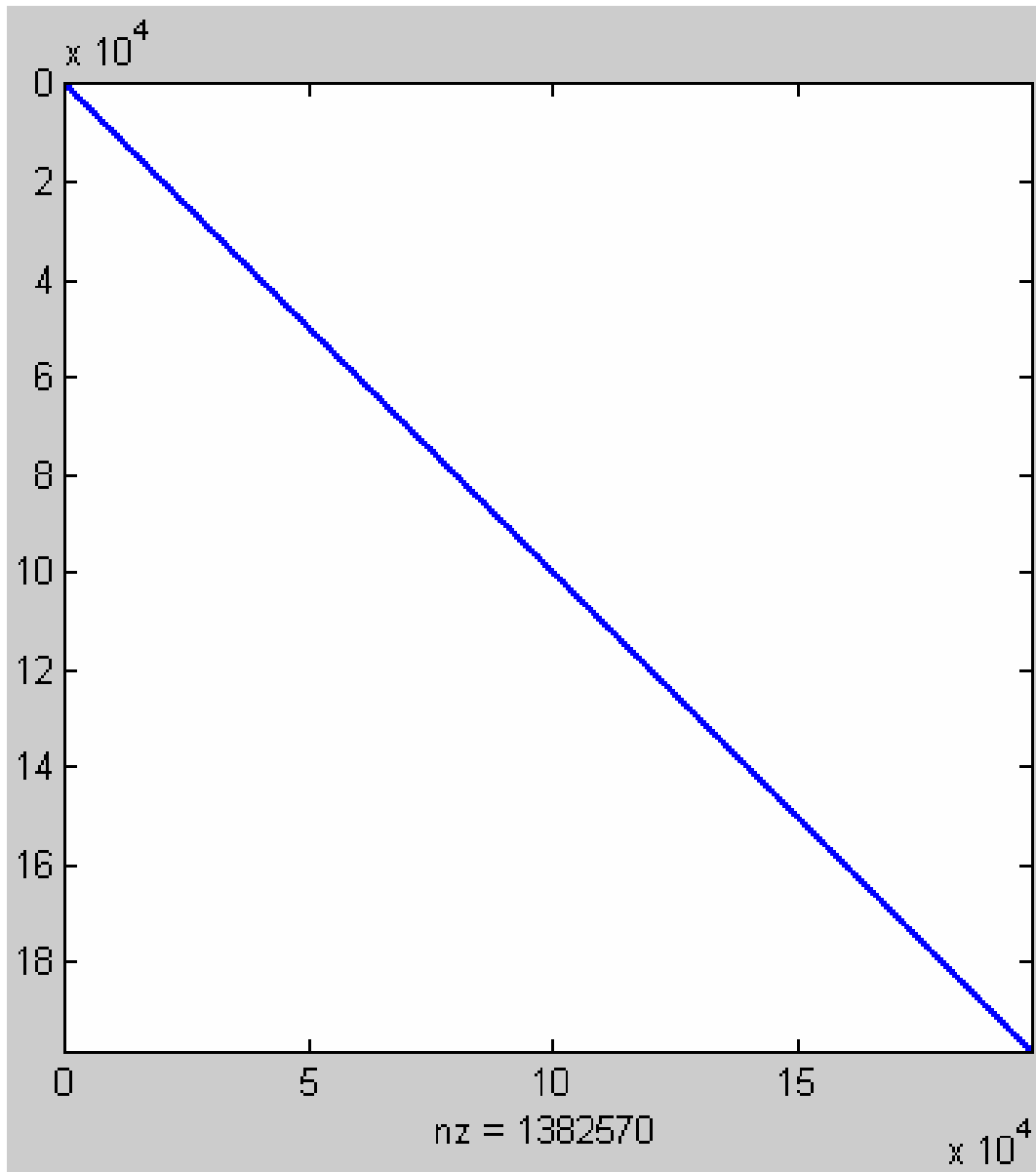# Arithmetic unit
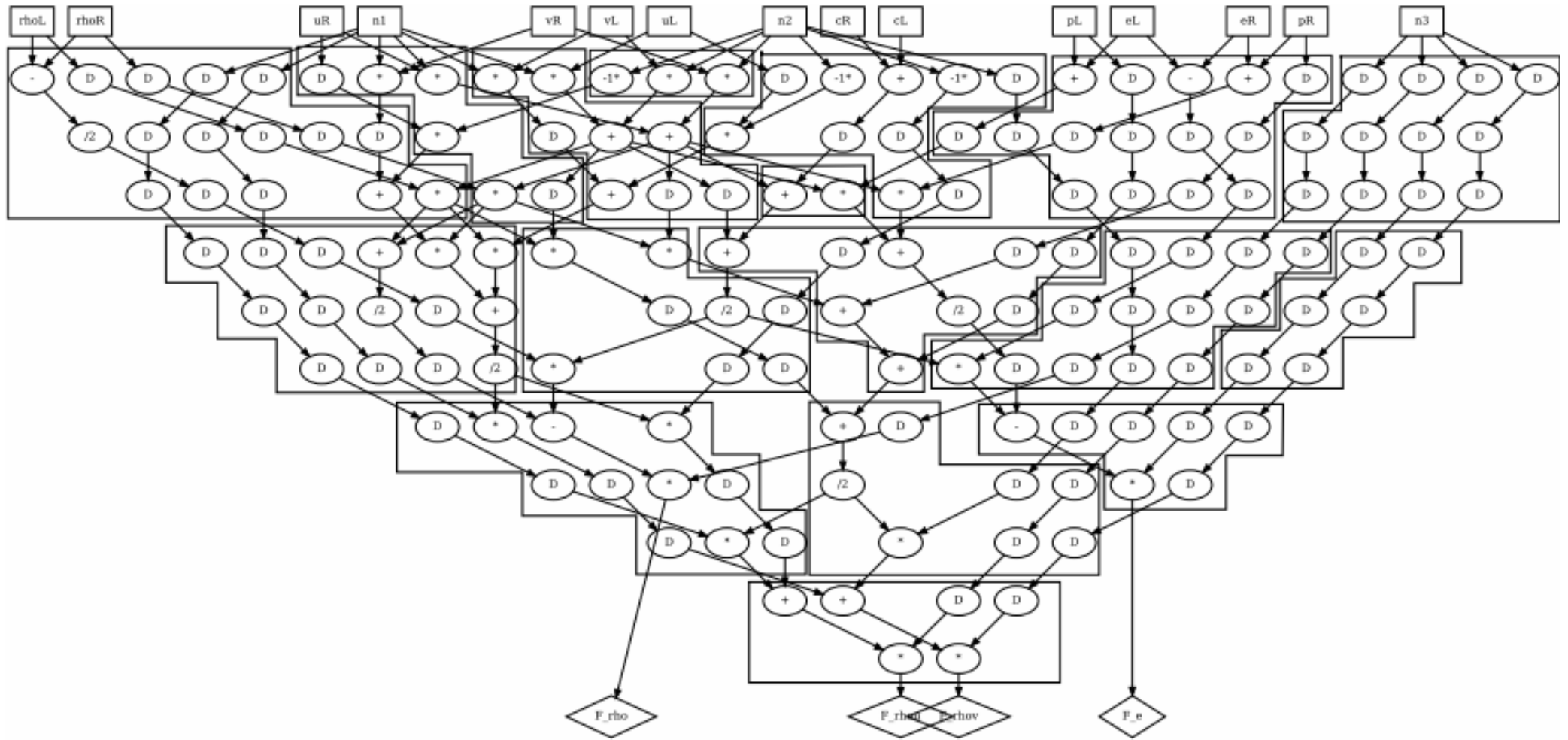
Results

Forward facing step
2D unstructured mesh

Adjacency Matrix:198,006 nodes


nz = 1382570

- Bandwidth: 198,006
- Memory:
  - 4 time dependent variables
  - 32 byte/cell, ~6MB
- Node degree: 3
  - 3 x 3byte adjacency list
  - 3 x 2 normal vector coordinate
  - 57 byte/cell, ~10.7MB
  - 3 clk/cell
  - 325MHz clock
  - memory bandwidth 23.5GB/s
  - nonuniform memory access pattern
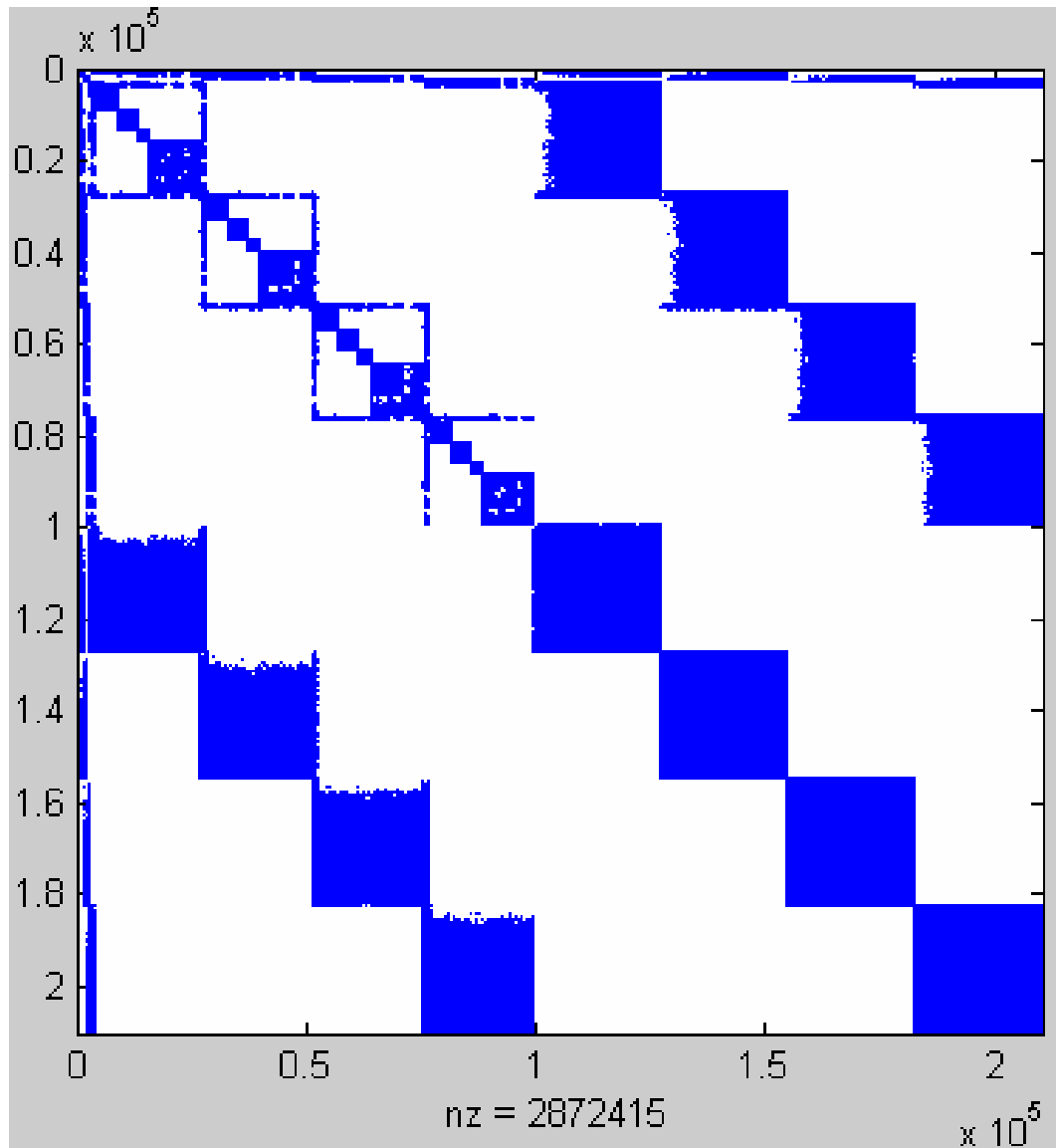
Renumbering

- Bandwidth: 580
- Memory requirements:
  - 1,160 cell
  - 32 byte/cell: ~36.2kB
- Node degree: 3
  - 3 x 2byte adjacency list
  - 3 x 2 normal vector coordinate
  - 54 byte/cell, ~61.1kB
  - 3 clk/cell
  - 325MHz clock
  - memory bandwidth 12.7GB/s
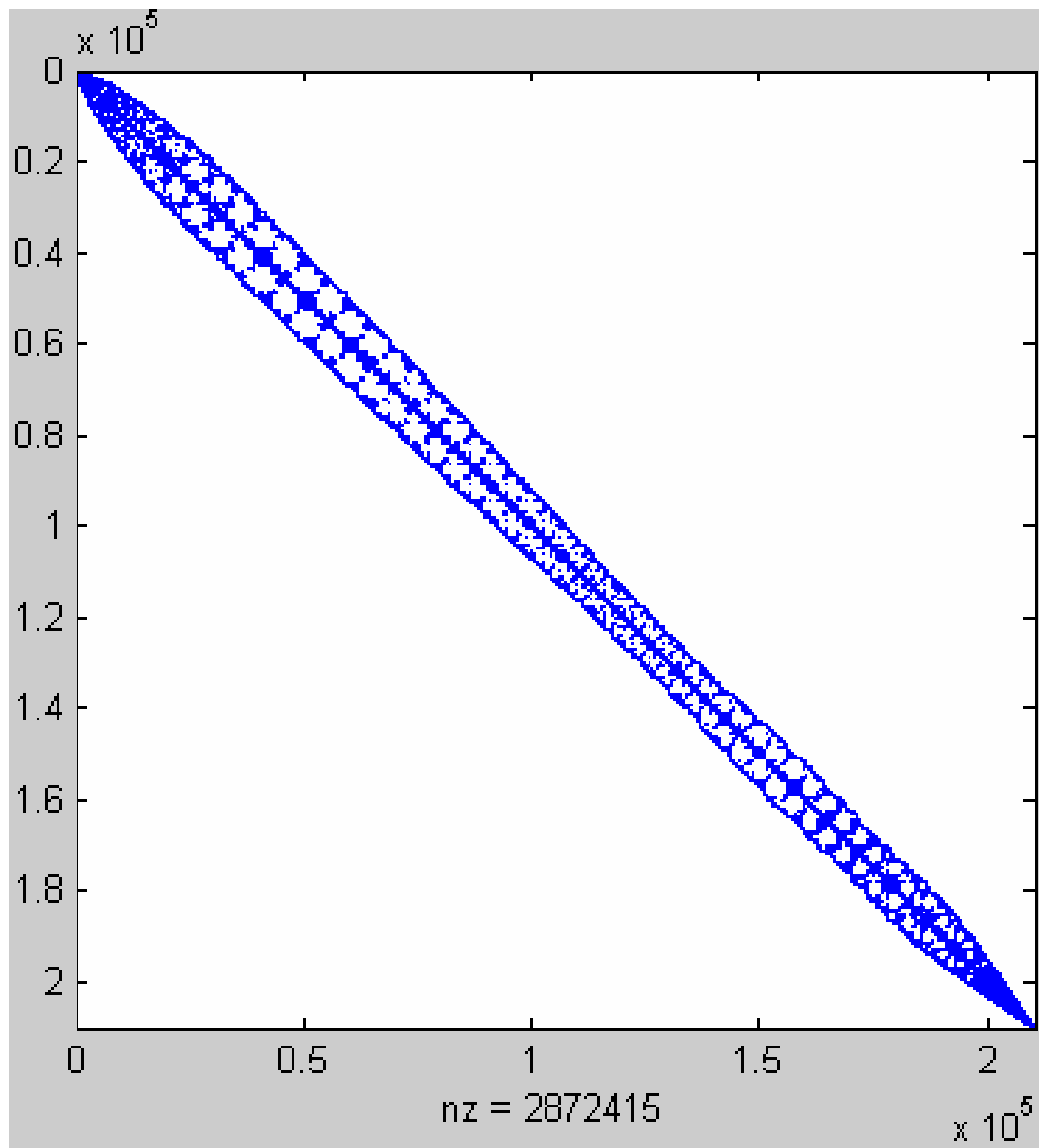
Scramjet 3D unstructured mesh
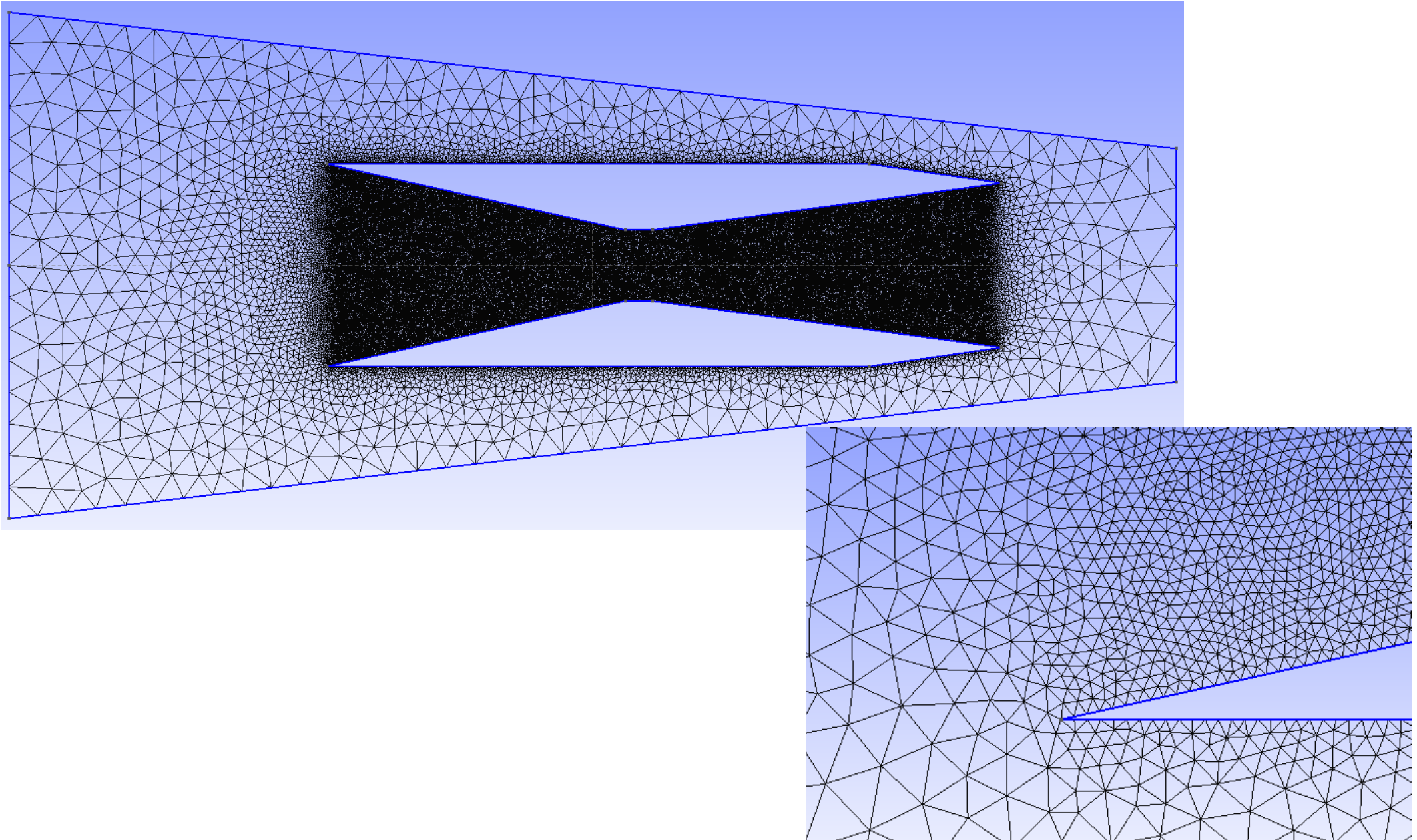
# Adjacency Matrix
Scramjet: 210,379 nodes

- Bandwidth: 210,379
- Memory:
  - 5 time dependent variables
  - 40 byte/cell, ~8MB
- Node degree: 4
  - 4 x 3byte adjacency list
  - 4 x 3 normal vector coordinate
  - 108 byte/cell, ~21.6MB
  - 4 clk/cell
  - 325MHz clock
  - memory bandwidth 28.2GB/s
  - nonuniform memory access pattern
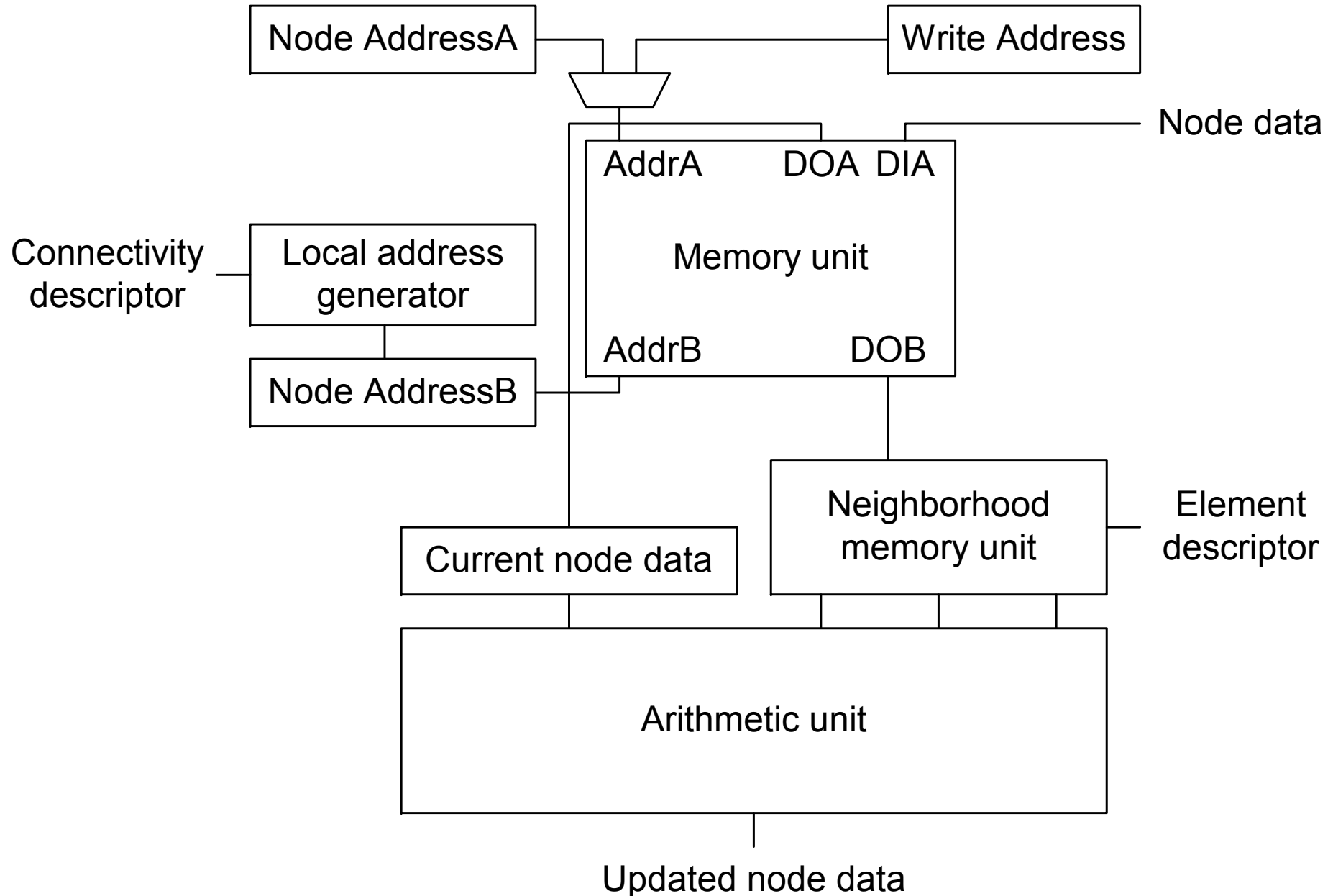
Renumbering

- Bandwidth: 10,317
- Memory requirements:
  - 20,634 cell
  - 40 byte/cell: ~806kB
- Node degree: 4
  - 4 x 2byte adjacency list
  - 4 x 3 normal vector coordinate
  - 104 byte/cell, ~2.04MB
  - 4 clk/cell
  - 325MHz clock
 - memory bandwidth 14.95GB/s

System Architecture
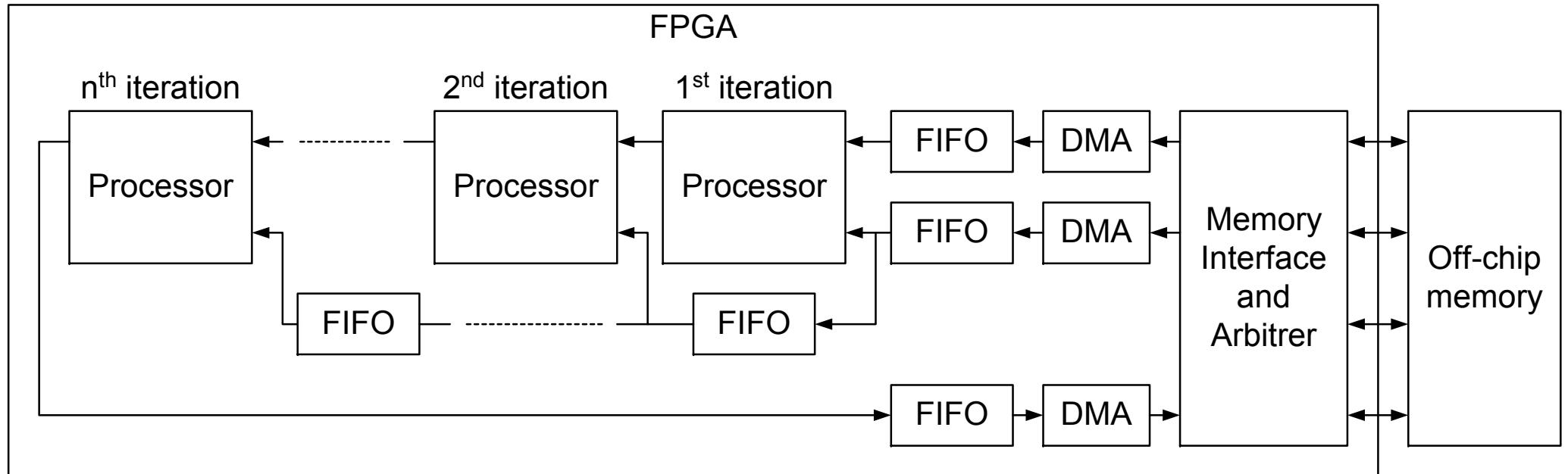
Processor Architecture

Performance

- Alpha-Data ADM-XRC-6T1
- FPGA: Xilinx XC6VSX475T
  - DSP: 525 (26%)
  - FF: 49,072 (12%)
  - LUT: 34,543 (8%)
  - 3 arithmetic units
- Clock frequency: 325MHz
- 325 million triangle update/s
- 69.22GFLOPs
- 76.3 times speedup

- Intel Xeon E5620 2.4GHz

Performance (Million triangle update/s) vs Number of triangles. Legend: unordered, reordered.
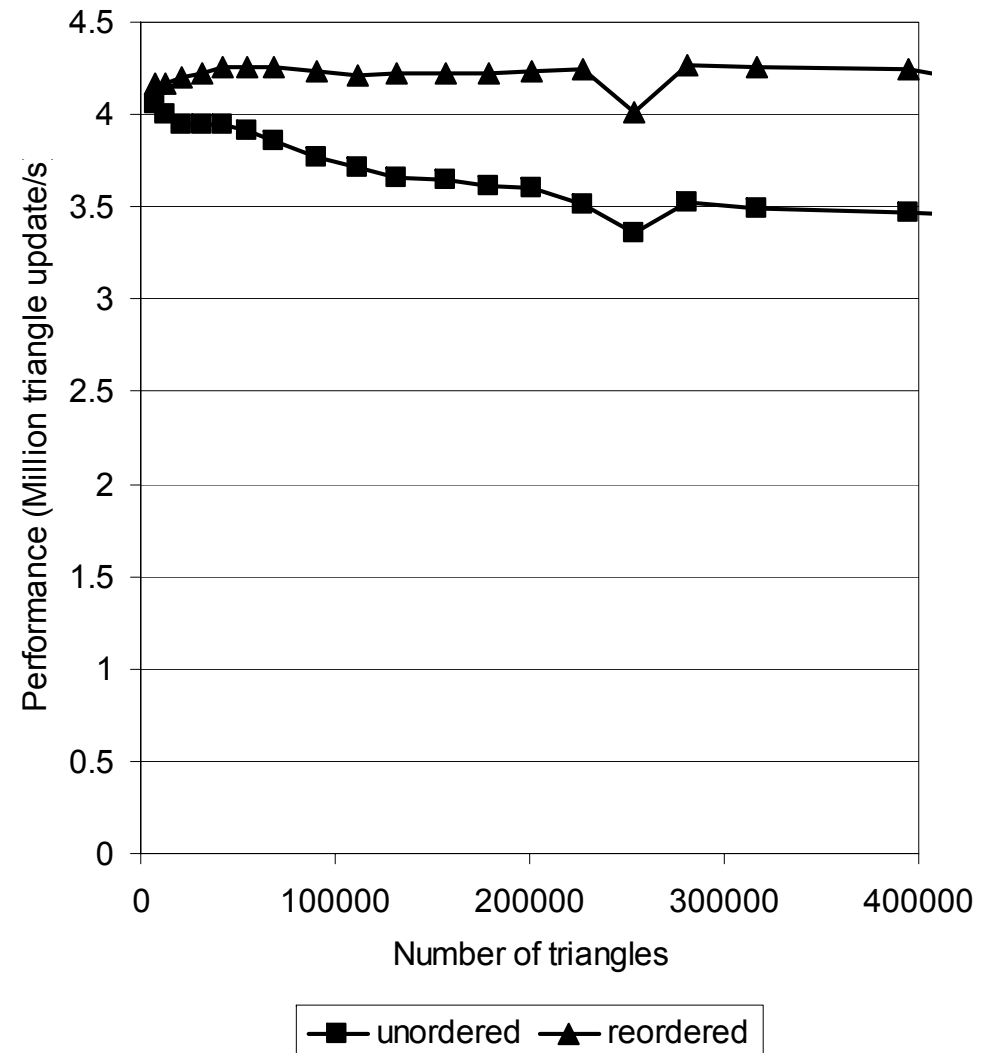
Conclusions, future work

- Supersonic flow simulation
  - High performance FPGA
  - Automatic arithmetic unit generation, partitioning, placement -> high clock frequency
  - Node reordering -> Efficient unstructured mesh handling
  - Nearly two orders (76.3 times) speedup
- Future work
  - Mesh partitioning
  - Multi FPGA

Example: 2D intersection of a Scramjet engine (1.4M grid points)